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RESEARCH ARTICLE

Emergent cosmology, inflation and dark energy

Eduardo Guendelman · Ramón Herrera · Pedro Labrana · Emil Nissimov · Svetlana Pacheva

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Abstract A new class of gravity–matter models defined in terms of two independent non-Riemannian volume forms (alternative generally covariant integration measure densities) on the space–time manifold are studied in some detail. These models involve an additional R^2 (square of the scalar curvature) term as well as scalar matter field potentials of appropriate form so that the pertinent action is invariant under global Weyl-scale symmetry. Scale invariance is spontaneously broken upon integration of the equations of motion for the auxiliary volume-form degrees of freedom. After performing transition to the physical Einstein frame we obtain: (1) an effective potential for the scalar field with two flat regions which allows for a unified description of both early universe inflation as well as of present dark energy epoch; (2) for a defi-

E. Guendelman (✉)
Department of Physics, Ben-Gurion University of the Negev, Beersheba, Israel
e-mail: guendel@bgu.ac.il; alexk@bgu.ac.il

R. Herrera
Instituto de Física, Pontificia Universidad Católica de Valparaíso,
Avenida Brasil 2950, Casilla, 4059 Valparaiso, Chile
e-mail: ramon.herrera@ucv.cl

P. Labrana
Departamento de Física, Facultad de Ciencias, Universidad del Bío-Bío,
Casilla 5-C, Concepción, Chile
e-mail: plabrana@ubiobio.cl

E. Nissimov · S. Pacheva
Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences,
Sofia, Bulgaria
e-mail: nissimov@inrne.bas.bg

S. Pacheva
e-mail: svetlana@inrne.bas.bg

nite parameter range the model possesses a non-singular "emergent universe" solution which describes an initial phase of evolution that precedes the inflationary phase; (3) for a reasonable choice of the parameters the present model conforms to the Planck Collaboration data.

Keywords Modified gravity theories · Non-Riemannian volume forms · Global Weyl-scale symmetry spontaneous breakdown · Flat regions of scalar potential · Non-singular origin of the universe

1 Introduction

Modern cosmology has been formulated in an attractive framework where many aspects of the observable universe can be incorporated. In this "standard cosmological" framework, the early universe (cf. the books [1–6] and references therein) starts with a period of exponential expansion called "inflation". In the inflationary period also primordial density perturbations are generated (Refs. [7,8] and references therein). The "inflation" is followed by particle creation, where the observed matter and radiation were generated [1–6], and finally the evolution arrives to a present phase of slowly accelerating universe [9–13]. In this standard model, however, at least two fundamental questions remain unanswered:

– The early inflation, although solving many cosmological puzzles, like the horizon and flatness problems, cannot address the initial singularity problem;
– There is no explanation for the existence of two periods of exponential expansion with such wildly different scales—the inflationary phase and the present phase of slowly accelerated expansion of the universe.

The best known mechanism for generating a period of accelerated expansion is through the presence of some vacuum energy. In the context of a scalar field theory, vacuum energy density appears naturally when the scalar field acquires an effective potential $U_{\rm eff}$ which has flat regions so that the scalar field can "slowly roll" [14–17] and its kinetic energy can be neglected resulting in an energy–momentum tensor $T_{\mu\nu} \simeq -g_{\mu\nu} U_{\rm eff}$.

The possibility of continuously connecting an inflationary phase to a slowly accelerating universe through the evolution of a single scalar field—the *quintessential inflation scenario*—has been first studied in Ref. [18]. Also, $F(R)$ models can yield both an early time inflationary epoch and a late time de Sitter phase with vastly different values of effective vacuum energies [19–21]. For a recent proposal of a quintessential inflation mechanism based on the k-essence [22–25] framework, see Ref. [26]. For another recent approach to quintessential inflation based on the "variable gravity" model [27] and for extensive list of references to earlier work on the topic, see Ref. [28].

In the present paper we will study a unified scenario where both an inflation and a slowly accelerated phase for the universe can appear naturally from the existence of two flat regions in the effective scalar field potential which we derive systematically from a Lagrangian action principle. Namely, we start with a new kind of globally Weyl-scale invariant gravity–matter action within the first-order (Palatini) approach formulated in terms of two different non-Riemannian volume forms (integration measures) [29]. In this new theory there is a single scalar field with kinetic terms coupled to both non-Riemannian measures, and in addition to the scalar curvature term R also an R^2 term is included (which is similarly allowed by global Weyl-scale invariance). Scale invariance is spontaneously broken upon solving part of the corresponding equations of motion due to the appearance of two arbitrary dimensionful integration constants. We find in the physical Einstein frame an effective k-essence [22–25] type of theory, where the effective scalar field potential has *two flat regions* corresponding to the two accelerating phases of the universe—the inflationary early universe and the present late universe.

In addition, within the flat region corresponding to the early universe we also obtain another phase that precedes the inflation and provides for a non-singular origin of the universe. It is of an "emergent universe" type [30–36], i.e., the universe starts as a static Einstein universe, the scalar field rolls with a constant speed through a flat region and there is a domain in the parameter space of the theory where such non-singular solution exists and is stable. To this end let us recall that the concept of "emergent universe" solves one of the principal puzzles in cosmology—the problem of initial singularity [37–39] including avoiding the singularity theorems for scalar field-driven inflationary cosmology [40,41].

Let us briefly recall the origin of current approach. The main idea comes from Refs. [42–45] (see also Refs. [46–50]), where some of us have proposed a new class of gravity–matter theories based on the idea that the action integral may contain a new metric-independent generally-covariant integration measure density, i.e., an alternative non-Riemannian volume form on the space–time manifold defined in terms of an auxiliary antisymmetric gauge field of maximal rank. The originally proposed modified-measure gravity–matter theories [42–50] contained two terms in the pertinent Lagrangian action—one with a non-Riemannian integration measure and a second one with the standard Riemannian integration measure (in terms of the square-root of the determinant of the Riemannian space–time metric). An important feature was the requirement for global Weyl-scale invariance which subsequently underwent dynamical spontaneous breaking [42,43]. The second action term with the standard Riemannian integration measure might also contain a Weyl-scale symmetry preserving R^2-term [45].

The latter formalism yields various new interesting results in all types of known generally covariant theories:

- (i) $D = 4$-dimensional models of gravity and matter fields containing the new measure of integration appear to be promising candidates for resolution of the dark energy and dark matter problems, the fifth force problem, and a natural mechanism for spontaneous breakdown of global Weyl-scale symmetry [42–50].
- (ii) Study of reparametrization invariant theories of extended objects (strings and branes) based on employing of a modified non-Riemannian world-sheet/world-volume integration measure [51] leads to dynamically induced variable string/brane tension and to string models of non-abelian confinement. Recently [52] this formalism was generalized to the case of string and brane models in curved supergravity background.

– (iii) Study in Refs. [53,54] of modified supergravity models with an alternative non-Riemannian volume form on the space–time manifold produces some outstanding new features: (a) This new formalism applied to minimal $N = 1$ supergravity naturally triggers the appearance of a dynamically generated cosmological constant as an arbitrary integration constant, which signifies a new explicit mechanism of spontaneous (dynamical) breaking of supersymmetry; (b) Applying the same formalism to anti-de Sitter supergravity allows us to appropriately choose the above mentioned arbitrary integration constant so as to obtain simultaneously a very small effective observable cosmological constant as well as a very large physical gravitino mass.

The plan of the present paper is as follows. In the next Sect. 2 we describe in some detail the general formalism for the new class of gravity–matter systems defined in terms of two independent non-Riemannian integration measures. In Sect. 3 we describe the properties of the two flat regions in the Einstein-frame effective scalar potential corresponding to the evolution of the early and late universe, respectively. In Sect. 4 we present a numerical analysis, for a reasonable choice of the parameters, of the resulting ratio of tensor-to-scalar perturbations and show that the present model conforms to the Planck Collaboration data. In Sect. 5 we derive a non-singular "emergent universe" solution of the new gravity–matter system. In Sect. 6 a numerical study of the transition between the emergent universe phase and the slow-roll inflationary phase via a short "super-inflation" period is given in some detail. We conclude in Sect. 7 with some discussions.

2 Gravity–matter formalism with two independent non-Riemannian volume-forms

We shall consider the following non-standard gravity–matter system with an action of the general form involving two independent non-Riemannian integration measure densities generalizing the model studied in [29] (for simplicity we will use units where the Newton constant is taken as $G_{\text{Newton}} = 1/16\pi$):

$$S = \int d^4x\, \Phi_1(A)\Big[R + L^{(1)}\Big] + \int d^4x\, \Phi_2(B)\Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}\Big]. \quad (1)$$

Here the following notations are used:

– $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms, i.e., generally covariant integration measure densities on the underlying space–time manifold:

$$\Phi_1(A) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda}, \quad \Phi_2(B) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda}, \quad (2)$$

defined in terms of field-strengths of two auxiliary 3-index antisymmetric tensor gauge fields[1]. $\Phi_{1,2}$ take over the role of the standard Riemannian integration measure density $\sqrt{-g} \equiv \sqrt{-\det \|g_{\mu\nu}\|}$ in terms of the space–time metric $g_{\mu\nu}$.
- $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ are the scalar curvature and the Ricci tensor in the first-order (Palatini) formalism, where the affine connection $\Gamma^{\mu}_{\nu\lambda}$ is *a priori* independent of the metric $g_{\mu\nu}$. Note that in the second action term we have added a R^2 gravity term (again in the Palatini form). Let us recall that $R + R^2$ gravity within the second order formalism (which was also the first inflationary model) was originally proposed in Ref. [55].
- $L^{(1,2)}$ denote two different Lagrangians of a single scalar matter field of the form (similar to the choice in Refs. [42,43]):

$$L^{(1)} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi), \quad V(\varphi) = f_1 \exp\{-\alpha\varphi\}, \tag{3}$$

$$L^{(2)} = -\frac{b}{2} e^{-\alpha\varphi} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + U(\varphi), \quad U(\varphi) = f_2 \exp\{-2\alpha\varphi\}, \tag{4}$$

where α, f_1, f_2 are dimensionful positive parameters, whereas b is a dimensionless one.
- $\Phi(H)$ indicates the dual field strength of a third auxiliary 3-index antisymmetric tensor gauge field:

$$\Phi(H) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu H_{\nu\kappa\lambda}, \tag{5}$$

whose presence is crucial for non-triviality of the model.

The scalar potentials have been chosen in such a way that the original action (1) is invariant under global Weyl-scale transformations:

$$\begin{aligned} &g_{\mu\nu} \to \lambda g_{\mu\nu}, \quad \Gamma^{\mu}_{\nu\lambda} \to \Gamma^{\mu}_{\nu\lambda}, \quad \varphi \to \varphi + \frac{1}{\alpha} \ln \lambda, \\ &A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa}, \quad B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa}, \quad H_{\mu\nu\kappa} \to H_{\mu\nu\kappa}. \end{aligned} \tag{6}$$

For the same reason we have multiplied by an appropriate exponential factor the scalar kinetic term in $L^{(2)}$ and also R and R^2 couple to the two different modified measures because of the different scalings of the latter.

Let us note that the requirement about the global Weyl-scale symmetry (6) uniquely fixes the structure of the non-Riemannian-measure gravity–matter action (1) (recall that the gravity terms R and R^2 are taken in the first order (Palatini) formalism).

Let us also note that the global Weyl-scale symmetry transformations defined in (6) are *not* the standard Weyl-scale (or conformal) symmetry known in ordinary conformal field theory. It is straightforward to check that the dimensionful parameters α, f_1, f_2 present in (3), (4) do *not* spoil at all the symmetry given in (6). In particular, unlike

[1] In D space–time dimensions one can always represent a maximal rank antisymmetric gauge field $A_{\mu_1 \ldots \mu_{D-1}}$ in terms of D auxiliary scalar fields ϕ^i $(i = 1, \ldots, D)$ in the form: $A_{\mu_1 \ldots \mu_{D-1}} = \frac{1}{D} \varepsilon_{i i_1 \ldots i_{D-1}} \phi^i \partial_{\mu_1} \phi^{i_1} \ldots \partial_{\mu_{D-1}} \phi^{i_{D-1}}$, so that its (dual) field-strength $\Phi(A) = \frac{1}{D!} \varepsilon_{i_1 \ldots i_D} \varepsilon^{\mu_1 \ldots \mu_D} \partial_{\mu_1} \phi^{i_1} \ldots \partial_{\mu_D} \phi^{i_D}$.

the standard form of the Weyl-scale transformation for the metric the transformation of the scalar field φ is not the canonical scale transformation known in standard conformal field theories. In fact, as shown in the second Ref. [43] in the context of a simpler than (1) model with only one non-Riemannian measure, upon appropriate φ-dependent conformal rescaling of the metric together with a scalar field redefinition $\varphi \to \phi \sim e^{-\varphi}$, one can transform the latter model into Zee's induced gravity model [56], where its pertinent scalar field ϕ transforms multiplicatively under the above scale transformations as in standard conformal field theory.

The equations of motion resulting from the action (1) are as follows. Variation of (1) w.r.t. affine connection $\Gamma^{\mu}_{\nu\lambda}$:

$$\int d^4 x \sqrt{-g} g^{\mu\nu} \Big(\frac{\Phi_1}{\sqrt{-g}} + 2\epsilon \frac{\Phi_2}{\sqrt{-g}} R \Big) \left(\nabla_\kappa \delta\Gamma^\kappa_{\mu\nu} - \nabla_\mu \delta\Gamma^\kappa_{\kappa\nu} \right) = 0 \qquad (7)$$

shows, following the analogous derivation in the Refs. [42,43], that $\Gamma^{\mu}_{\nu\lambda}$ becomes a Levi-Civita connection:

$$\Gamma^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\nu\lambda}(\bar{g}) = \frac{1}{2} \bar{g}^{\mu\kappa} \left(\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda} \right), \qquad (8)$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon \chi_2 R) g_{\mu\nu}, \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}}, \quad \chi_2 \equiv \frac{\Phi_2(B)}{\sqrt{-g}}. \qquad (9)$$

Variation of the action (1) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu \Big[R + L^{(1)} \Big] = 0, \qquad \partial_\mu \Big[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} \Big] = 0, \qquad \partial_\mu \Big(\frac{\Phi_2(B)}{\sqrt{-g}} \Big) = 0, \quad (10)$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const}, \quad R + L^{(1)} = -M_1 = \text{const}$$
$$L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const}. \qquad (11)$$

Here M_1 and M_2 are arbitrary dimensionful and χ_2 arbitrary dimensionless integration constants.

The first integration constant χ_2 in (11) preserves global Weyl-scale invariance (6), whereas the appearance of the second and third integration constants M_1, M_2 signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance under (6) due to the scale non-invariant solutions (second and third ones) in (11).

To this end let us recall that classical solutions of the whole set of equations of motion (not only those of the scalar field(s)) correspond in the semiclassical limit to ground-state expectation values of the corresponding fields. In the present case some of the pertinent classical solutions (second and third Eqs. (11)) contain arbitrary integration constants $M_1,\ M_2$ whose appearance makes these solutions non-covariant w.r.t. the symmetry transformations (6). Thus, spontaneous symmetry breaking of (6) is not necessarily originating from some fixed extrema of the scalar potentials. In fact, as we will see in the next Section below, the (static) classical solutions for the scalar field defined through extremizing the effective Einstein-frame scalar potential (Eq. (27) below) belong to the two infinitely large flat regions of the latter (infinitely large "valleys" of "ground states"), therefore, this does not constitute a breakdown of the shift symmetry of the scalar field (6). Thus, it is the appearance of the arbitrary integration constants $M_1,\ M_2$, which triggers the spontaneous breaking of global Weyl-scale symmetry (6).

Varying (1) w.r.t. $g_{\mu\nu}$ and using relations (11) we have:

$$\chi_1\Big[R_{\mu\nu} + \frac{1}{2}\left(g_{\mu\nu}L^{(1)} - T^{(1)}_{\mu\nu}\right)\Big] - \frac{1}{2}\chi_2\Big[T^{(2)}_{\mu\nu} + g_{\mu\nu}\left(\epsilon R^2 + M_2\right) - 2R\,R_{\mu\nu}\Big] = 0\,, \tag{12}$$

where χ_1 and χ_2 are defined in (9), and $T^{(1,2)}_{\mu\nu}$ are the energy–momentum tensors of the scalar field Lagrangians with the standard definitions:

$$T^{(1,2)}_{\mu\nu} = g_{\mu\nu}L^{(1,2)} - 2\frac{\partial}{\partial g^{\mu\nu}}L^{(1,2)}. \tag{13}$$

Taking the trace of Eqs. (12) and using again second relation (11) we solve for the scale factor χ_1:

$$\chi_1 = 2\chi_2\frac{T^{(2)}/4 + M_2}{L^{(1)} - T^{(1)}/2 - M_1}\,, \tag{14}$$

where $T^{(1,2)} = g^{\mu\nu}T^{(1,2)}_{\mu\nu}$.

Using second relation (11) Eqs. (12) can be put in the Einstein-like form:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}g_{\mu\nu}\left(L^{(1)} + M_1\right) + \frac{1}{2\Omega}\left(T^{(1)}_{\mu\nu} - g_{\mu\nu}L^{(1)}\right) \\ + \frac{\chi_2}{2\chi_1\Omega}\Big[T^{(2)}_{\mu\nu} + g_{\mu\nu}\left(M_2 + \epsilon(L^{(1)} + M_1)^2\right)\Big], \end{aligned} \tag{15}$$

where:

$$\Omega = 1 - \frac{\chi_2}{\chi_1}2\epsilon\left(L^{(1)} + M_1\right). \tag{16}$$

Let us note that (9), upon taking into account second relation (11) and (16), can be written as:

$$\bar{g}_{\mu\nu} = \chi_1\Omega\, g_{\mu\nu}. \tag{17}$$

Now, we can bring Eqs. (15) into the standard form of Einstein equations for the rescaled metric $\bar{g}_{\mu\nu}$ (17), i.e., the Einstein-frame gravity equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}T^{\rm eff}_{\mu\nu} \tag{18}$$

with energy–momentum tensor corresponding (according to (13)):

$$T^{\rm eff}_{\mu\nu} = g_{\mu\nu} L_{\rm eff} - 2\frac{\partial}{\partial g^{\mu\nu}} L_{\rm eff} \tag{19}$$

to the following effective Einstein-frame scalar field Lagrangian:

$$L_{\rm eff} = \frac{1}{\chi_1 \Omega}\Big\{L^{(1)} + M_1 + \frac{\chi_2}{\chi_1 \Omega}\Big[L^{(2)} + M_2 + \epsilon (L^{(1)} + M_1)^2\Big]\Big\}. \tag{20}$$

In order to explicitly write $L_{\rm eff}$ in terms of the Einstein-frame metric $\bar{g}_{\mu\nu}$ (17) we use the short-hand notation for the scalar kinetic term:

$$X \equiv -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \tag{21}$$

and represent $L^{(1,2)}$ in the form:

$$L^{(1)} = \chi_1 \Omega\, X - V, \quad L^{(2)} = \chi_1 \Omega\, b e^{-\alpha\varphi} X + U, \tag{22}$$

with V and U as in (3), (4).

From Eqs. (14) and (16), taking into account (22), we find:

$$\frac{1}{\chi_1 \Omega} = \frac{(V - M_1)}{2\chi_2\Big[U + M_2 + \epsilon (V - M_1)^2\Big]}\Big[1 - \chi_2\Big(\frac{b e^{-\alpha\varphi}}{V - M_1} - 2\epsilon\Big)X\Big]. \tag{23}$$

Upon substituting expression (23) into (20) we arrive at the explicit form for the Einstein-frame scalar Lagrangian:

$$L_{\rm eff} = A(\varphi) X + B(\varphi) X^2 - U_{\rm eff}(\varphi), \tag{24}$$

where:

$$\begin{aligned} A(\varphi) &\equiv 1 + \Big[\frac{1}{2} b e^{-\alpha\varphi} - \epsilon (V - M_1)\Big]\frac{V - M_1}{U + M_2 + \epsilon (V - M_1)^2} \\ &= 1 + \Big[\frac{1}{2} b e^{-\alpha\varphi} - \epsilon \left(f_1 e^{-\alpha\varphi} - M_1\right)\Big]\frac{f_1 e^{-\alpha\varphi} - M_1}{f_2 e^{-2\alpha\varphi} + M_2 + \epsilon (f_1 e^{-\alpha\varphi} - M_1)^2}, \end{aligned} \tag{25}$$

and

$$\begin{aligned} B(\varphi) &\equiv \chi_2 \frac{\epsilon\Big[U + M_2 + (V - M_1) b e^{-\alpha\varphi}\Big] - \frac{1}{4} b^2 e^{-2\alpha\varphi}}{U + M_2 + \epsilon (V - M_1)^2} \\ &= \chi_2 \frac{\epsilon\Big[f_2 e^{-2\alpha\varphi} + M_2 + (f_1 e^{-\alpha\varphi} - M_1) b e^{-\alpha\varphi}\Big] - \frac{1}{4} b^2 e^{-2\alpha\varphi}}{f_2 e^{-2\alpha\varphi} + M_2 + \epsilon (f_1 e^{-\alpha\varphi} - M_1)^2}, \end{aligned} \tag{26}$$

whereas the effective scalar field potential reads:

$$U_{\text{eff}}(\varphi) \equiv \frac{(V - M_1)^2}{4\chi_2\Big[U + M_2 + \epsilon(V - M_1)^2\Big]} = \frac{\big(f_1 e^{-\alpha\varphi} - M_1\big)^2}{4\chi_2\Big[f_2 e^{-2\alpha\varphi} + M_2 + \epsilon(f_1 e^{-\alpha\varphi} - M_1)^2\Big]}, \tag{27}$$

where the explicit form of V and U (3), (4) are inserted.

Let us recall that the dimensionless integration constant χ_2 is the ratio of the original second non-Riemannian integration measure to the standard Riemannian one (9).

To conclude this Section let us note that choosing the "wrong" sign of the scalar potential $U(\varphi)$ (Eq. (4)) in the initial non-Riemannian-measure gravity–matter action (1) is necessary to end up with the right sign in the effective scalar potential (27) in the physical Einstein-frame effective gravity–matter action (24). On the other hand, the overall sign of the other initial scalar potential $V(\varphi)$ (Eq. (4)) is in fact irrelevant since changing its sign does not affect the positivity of effective scalar potential (27).

Let us also remark that the effective matter Lagrangian (24) is called "Einstein-frame scalar Lagrangian" in the sense that it produces the effective energy–momentum tensor (19) entering the effective Einstein-frame form of the gravity equations of motion (18) in terms of the conformally rescaled metric $\bar{g}_{\mu\nu}$ (17) which have the canonical form of Einstein's gravitational equations. On the other hand, the pertinent Einstein-frame effective scalar Lagrangian (24) arises in a non-canonical "k-essence" [22–25] type form.

3 Flat regions of the effective scalar potential

Depending on the sign of the integration constant M_1 we obtain two types of shapes for the effective scalar potential $U_{\text{eff}}(\varphi)$ (27) depicted on Figs. 1 and 2.

Due to the vast difference in the scales of the pertinent parameters, whose estimates are given below, Figs. 1 and 2 represent only qualitatively the shape of $U_{\text{eff}}(\varphi)$.

The crucial feature of $U_{\text{eff}}(\varphi)$ is the presence of two infinitely large flat regions—for large negative and large positive values of the scalar field φ. For large negative values of φ we have for the effective potential and the coefficient functions in the Einstein-frame scalar Lagrangian (24)–(27):

$$U_{\text{eff}}(\varphi) \simeq U_{(-)} \equiv \frac{f_1^2/f_2}{4\chi_2(1 + \epsilon f_1^2/f_2)}, \tag{28}$$

$$A(\varphi) \simeq A_{(-)} \equiv \frac{1+\frac{1}{2}bf_1/f_2}{1+\epsilon f_1^2/f_2}, \quad B(\varphi) \simeq B_{(-)} \equiv -\chi_2 \frac{b^2/4f_2 - \epsilon(1+bf_1/f_2)}{1+\epsilon f_1^2/f_2}. \tag{29}$$

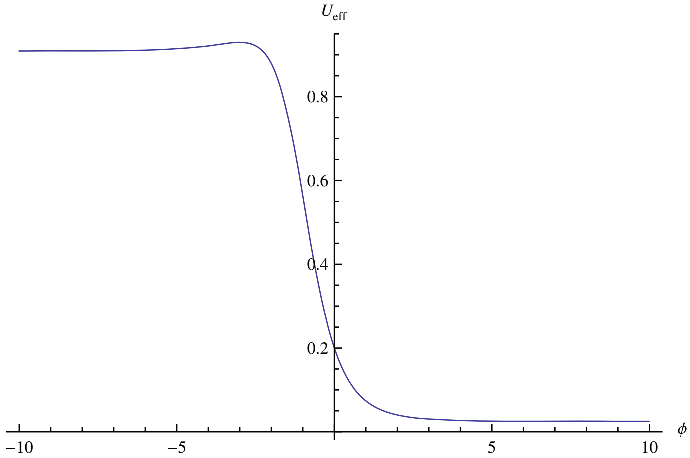

Fig. 1 Qualitative shape of the effective scalar potential $U_{\rm eff}$ (27) as function of φ for $M_1 < 0$

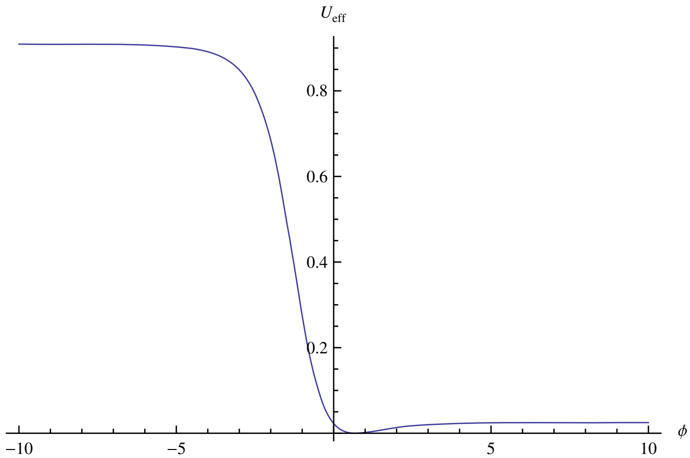

Fig. 2 Qualitative shape of the effective scalar potential $U_{\rm eff}$ (27) as function of φ for $M_1 > 0$

In the second flat region for large positive φ:

$$U_{\rm eff}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2(1+\epsilon M_1^2/M_2)}, \tag{30}$$

$$A(\varphi) \simeq A_{(+)} \equiv \frac{M_2}{M_2+\epsilon M_1^2}, \quad B(\varphi) \simeq B_{(+)} \equiv \epsilon\chi_2 \frac{M_2}{M_2+\epsilon M_1^2}. \tag{31}$$

From the expression for $U_{\rm eff}(\varphi)$ (27) and the Figs. 1 and 2 we see that now we have an explicit realization of quintessential inflation scenario. The flat regions (28), (29) and (30), (31) correspond to the evolution of the early and the late universe, respectively, provided we choose the ratio of the coupling constants in the original scalar potentials versus the ratio of the scale-symmetry breaking integration constants to obey:

$$\frac{f_1^2/f_2}{1+\epsilon f_1^2/f_2} \gg \frac{M_1^2/M_2}{1+\epsilon M_1^2/M_2}\,, \tag{32}$$

which makes the vacuum energy density of the early universe $U_{(-)}$ much bigger than that of the late universe $U_{(+)}$ (cf. (28), (30)). The inequality (32) is equivalent to the requirements:

$$\frac{f_1^2}{f_2} \gg \frac{M_1^2}{M_2}, \quad |\epsilon|\frac{M_1^2}{M_2} \ll 1. \tag{33}$$

In particular, if we choose the scales of the scale symmetry breaking integration constants $|M_1| \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$, where M_{EW}, M_{Pl} are the electroweak and Planck scales, respectively, we are then naturally led to a very small vacuum energy density $U_{(+)} \sim M_1^2/M_2$ of the order:

$$U_{(+)} \sim M_{EW}^8/M_{Pl}^4 \sim 10^{-120} M_{Pl}^4, \tag{34}$$

which is the right order of magnitude for the present epoche's vacuum energy density as already recognized in Ref. [58]. On the other hand, if we take the order of magnitude of the coupling constants in the effective potential $f_1 \sim f_2 \sim (10^{-2} M_{Pl})^4$, then together with the above choice of order of magnitudes for $M_{1,2}$ the inequalities (33) will be satisfied as well and the order of magnitude of the vacuum energy density of the early universe $U_{(-)}$ (28) becomes:

$$U_{(-)} \sim f_1^2/f_2 \sim 10^{-8} M_{Pl}^4, \tag{35}$$

which conforms to the BICEP2 experiment [59] and Planck Collaboration data [60,61] implying the energy scale of inflation of order $10^{-2} M_{Pl}$. However, let us remark at this point that, as shown in the next Sect. 4, the result for the tensor-to-scalar ratio r obtained within the present model conforms to the data of the Planck Collaboration [60,61] rather than BICEP2 [59].

Let us recall that, since we are using units where $G_{\text{Newton}} = 1/16\pi$, in the present case $M_{Pl} = \sqrt{1/8\pi G_{\text{Newton}}} = \sqrt{2}$.

Before proceeding to the derivation of the non-singular "emergent universe" solution describing an initial phase of the universe evolution preceding the inflationary phase, let us briefly sketch how the present non-Riemannian-measure-modified gravity–matter theory meets the conditions for the validity of the "slow-roll" approximation [14,15] when φ evolves on the flat region of the effective potential corresponding to the early universe (28), (29).

To this end let us recall the standard Friedman–Lemaitre–Robertson–Walker space–time metric [57]:

$$ds^2 = -dt^2 + a^2(t)\Big[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big] \tag{36}$$

and the associated Friedman equations (recall the presently used units $G_{\text{Newton}} = 1/16\pi$):

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p), \quad H^2 + \frac{K}{a^2} = \frac{1}{6}\rho, \quad H \equiv \frac{\dot{a}}{a}, \tag{37}$$

describing the universe' evolution. Here:

$$\rho = \frac{1}{2}A(\varphi)\dot{\varphi}^2 + \frac{3}{4}B(\varphi)\dot{\varphi}^4 + U_{\text{eff}}(\varphi), \tag{38}$$

$$p = \frac{1}{2}A(\varphi)\dot{\varphi}^2 + \frac{1}{4}B(\varphi)\dot{\varphi}^4 - U_{\text{eff}}(\varphi) \tag{39}$$

are the energy density and pressure of the scalar field $\varphi = \varphi(t)$. Henceforth the dots indicate derivatives with respect to the time t.

Let us now consider the standard "slow-roll" parameters [16,17]:

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \eta \equiv -\frac{\ddot{\varphi}}{H\dot{\varphi}}, \tag{40}$$

where ε measures the ratio of the scalar field kinetic energy relative to its total energy density and η measures the ratio of the field's acceleration relative to the "friction" ($\sim 3H\dot{\varphi}$) term in the pertinent scalar field equations of motion:

$$\ddot{\varphi}(A + 3B\dot{\varphi}^2) + 3H\dot{\varphi}(A + B\dot{\varphi}^2) + U'_{\text{eff}} + \frac{1}{2}A'\dot{\varphi}^2 + \frac{3}{4}B'\dot{\varphi}^4 = 0, \tag{41}$$

with primes indicating derivatives w.r.t. φ.

In the slow-roll approximation one ignores the terms with $\ddot{\varphi}$, $\dot{\varphi}^2$, $\dot{\varphi}^3$, $\dot{\varphi}^4$ so that the φ-equation of motion (41) and the second Friedman Eq. (37) reduce to:

$$3AH\dot{\varphi} + U'_{\text{eff}} = 0, \quad H^2 = \frac{1}{6}U_{\text{eff}}. \tag{42}$$

The reason for ignoring the spatial curvature term K/a^2 in the second Eq. (42) is due to the fact that φ evolves on a flat region of U_{eff} and the Hubble parameter $H \equiv \dot{a}/a \simeq \text{const}$, so that $a(t)$ grows exponentially with time making K/a^2 very small. Consistency of the slow-roll approximation implies for the slow-roll parameters (40), taking into account (42), the following inequalities:

$$\varepsilon \simeq \frac{1}{A}\left(\frac{U'_{\text{eff}}}{U_{\text{eff}}}\right)^2 \ll 1, \quad \eta \simeq \frac{2}{A}\frac{U''_{\text{eff}}}{U_{\text{eff}}} - \varepsilon - \frac{2A'}{A^{3/2}}\sqrt{\varepsilon} \;\; \rightarrow \;\; \frac{2}{A}\frac{U''_{\text{eff}}}{U_{\text{eff}}} \ll 1. \tag{43}$$

Since now φ evolves on the flat region of U_{eff} for large negative values (28), the Lagrangian coefficient function $A(\varphi) \simeq A_{(-)}$ as in (29) and the gradient of the effective scalar potential is:

$$U'_{\text{eff}} \simeq -\frac{\alpha f_1 M_1 e^{\alpha\varphi}}{2\chi_2 f_2(1 + \epsilon f_1^2/f_2)^2}, \tag{44}$$

which yields for the slow-roll parameter ε (43):

$$\varepsilon \simeq \frac{4\alpha^2 M_1^2 e^{2\alpha\varphi}}{f_1^2(1+bf_1/2f_2)(1+\epsilon f_1^2/f_2)} \ll 1 \quad \text{for large negative } \varphi. \tag{45}$$

Similarly, for the second slow-roll parameter we have:

$$\left|\frac{2}{A}\frac{U''_{\text{eff}}}{U_{\text{eff}}}\right| \simeq \frac{4\alpha^2|M_1|e^{\alpha\varphi}}{f_1(1+bf_1/2f_2)} \ll 1 \quad \text{for large negative } \varphi. \tag{46}$$

At this point let us remark that the non-canonical "k-essence" form of the effective scalar Lagrangian (24) does not affect the condition for smallness of the standard "slow-roll" parameters (40). Indeed, the definition of the first slow-roll parameter ε in (40) is consistent with the first Friedman equation in (37), where there is no *a priori* requirement for the energy density and the pressure to be defined in terms of a scalar field action with a canonical kinetic term. Similarly, the non-canonical "k-essence" form of the effective scalar Lagrangian (24) does not affect the requirement for smallness of the second "slow-roll" parameter η in (40). In fact, the smallness of ϵ, η is explicitly displayed in Eqs. (45), (46) because of the presence of strongly suppressing factors—exponentials of large negative values of the scalar field in the first flat region of the effective scalar potential corresponding to the early universe.

The value of φ at the end of the slow-roll regime φ_{end} is determined from the condition $\varepsilon \simeq 1$ which through (45) yields:

$$e^{-2\alpha\varphi_{\text{end}}} \simeq \frac{4\alpha^2 M_1^2}{f_1^2(1+bf_1/2f_2)(1+\epsilon f_1^2/f_2)}. \tag{47}$$

The number of *e-foldings* N (see, e.g. second Ref. [8]) between two values of cosmological times t_* and t_{end} or analogously between two different values φ_* and φ_{end} becomes:

$$N = \int_{t_*}^{t_{\text{end}}} H dt = \int_{\varphi_*}^{\varphi_{\text{end}}} \frac{H}{\dot\varphi} d\varphi \simeq -\int_{\varphi_*}^{\varphi_{\text{end}}} \frac{3H^2 A}{U'_{\text{eff}}} d\varphi \simeq -\int_{\varphi_*}^{\varphi_{\text{end}}} \frac{AU_{\text{eff}}}{2U'_{\text{eff}}} d\varphi, \tag{48}$$

where Eq. (42) are used. Substituting (28), (29) and (44) into (48) yields an expression for N which together with (47) allows for the determination of φ_*:

$$N \simeq \frac{f_1(1+bf_1/f_2)}{4\alpha^2 M_1}\left(e^{-\alpha\varphi_*} - e^{-\alpha\varphi_{\text{end}}}\right). \tag{49}$$

In what follows the subscript $*$ is used to indicate the epoch where the cosmological scale exits the horizon.

4 Perturbations

In the following we will describe the scalar and tensor perturbations for our model. Following Refs. [62–64] the power spectrum of the scalar perturbation $\mathcal{P}_S$ for a non-canonical kinetic term in the slow-roll approximation is given by:

$$\mathcal{P}_S = k_1 \frac{H^2}{c_s\,\varepsilon_1}, \tag{50}$$

where c_s denotes the "speed of sound" and is defined as $c_s^2 = \frac{P_{,X}}{P_{,X}+2XP_{,XX}}$, and $\varepsilon_1 = X\,P_{,X}/(16\pi^2\,H^2)$. Here $P(X,\varphi)$ is a function of the scalar field φ and X is the scalar kinetic term as in (21). The constant $k_1 = (G_{Newton}/8\pi^2) = (16\times 8\pi^3)^{-1}$, and $P_{,X}$ denotes the derivative with respect X. In particular, in the present case $P(X,\varphi) = L_{\text{eff}} = A(\varphi)\,X + B(\varphi)\,X^2 - U_{\text{eff}}(\varphi)$ (24) where $X = \dot{\varphi}^2/2$.

The scalar spectral index n_s is given by:

$$n_s - 1 = \frac{d\ln\mathcal{P}_S}{d\ln k} = -2\varepsilon_1 - \varepsilon_2 - \varepsilon_3, \tag{51}$$

where the parameters ε_2 and ε_3 are defined as $\varepsilon_2 = \frac{\dot{\varepsilon}_1}{\varepsilon_1\,H}$ and $\varepsilon_3 = \frac{\dot{c}_s}{c_s\,H}$, respectively [62–64].

On the other hand, it is well known that the generation of tensor perturbations during inflation would generate gravitational waves. The spectrum of the tensor perturbations $\mathcal{P}_T$ was calculated in Ref. [62] and is given by:

$$\mathcal{P}_T = \frac{2}{3\pi^2}\left(\frac{2XP_{,X} - P}{(16\pi)^2}\right), \tag{52}$$

and the tensor spectral index n_T can be expressed in terms of the parameter ε_1 as $n_T = \frac{d\ln\mathcal{P}_T}{d\ln k} = -2\varepsilon_1$. An important observational quantity is the tensor-to-scalar ratio $r = \frac{\mathcal{P}_T}{\mathcal{P}_S}$ satisfying a generalized consistency relation in which $r = -8\,c_s\,n_T$. These observational quantities should be evaluated at $\varphi = \varphi_*$ (see Eq. (49)).

Considering the slow-roll approximation the power spectrum of the scalar perturbation $\mathcal{P}_S$ (to leading order) from Eq. (50) becomes:

$$\mathcal{P}_S \simeq k_1\,C_1\,e^{-2\alpha\varphi_*}, \tag{53}$$

where the constant C_1 is given by

$$C_1 = \frac{(1+\frac{bf_1}{2f_2})}{16\chi_2\alpha^2\,M_1^2}\,\frac{f_1^4}{18\,f_2}.$$

From Eq. (51) the scalar spectral index n_s, becomes:

$$n_s \simeq 1 - \frac{2\alpha^2 M_1}{f_1(1+\epsilon f_1^2/f_2)}\,e^{\alpha\varphi_*} - \frac{4\alpha^2 M_1^2}{f_1^2(1+\epsilon f_1^2/f_2)}\left[\frac{1}{4\pi^2(1+bf_1/2f_2)}+1\right]e^{2\alpha\varphi_*}. \tag{54}$$

Combining Eqs. (45) and (54) the scalar spectral index n_s can be expresses in terms of the number of e-foldings N (48) to give:

$$n_s \simeq 1 - \frac{\alpha(1+bf_1/2f_2)}{(1+\epsilon f_1^2/f_2)}[C_2 + 2\alpha N]^{-1} - \frac{(1+bf_1/2f_2)^2}{(1+\epsilon f_1^2/f_2)}\left[\frac{1}{4\pi^2(1+bf_1/2f_2)} + 1\right][C_2 + 2\alpha N]^{-2}. \quad (55)$$

Here we took into account the relation between φ_* and the number of e-foldings N (49), which can be written as:

$$e^{\alpha\varphi_*} = \frac{f_1(1+bf_1/2f_2)}{2\alpha M_1}[C_2 + 2\alpha N]^{-1}, \quad (56)$$

where the constant C_2 is given by:

$$C_2 = \sqrt{\frac{(1+bf_1/2f_2)}{(1+\epsilon f_1^2/f_2)}}. \quad (57)$$

From Eqs. (53) and (56) we can write the parameter χ_2 in terms of the number of e-folds N and the power spectrum as:

$$\chi_2 = \frac{k_1 f_1^2 [C_2 + 2\alpha N]^2}{72 f_2(1+bf_1/2f_2)} \frac{1}{\mathcal{P}_S}. \quad (58)$$

In this form, we can obtain the value of the parameter χ_2 for given values f_1, f_2,b, ϵ and α parameters when the number of e-folds N and the power spectrum $\mathcal{P}_S$ are given.

From Eq. (55) and considering that $r = 16 c_s \varepsilon_1$, the relation between the tensor-to-scalar ratio r and the spectral index n_s , i.e., the consistency relation, $n_s = n_s(r)$, is given by:

$$n_s \simeq 1 - \frac{\pi\,\alpha\,C_2}{\sqrt{2}}\,r^{1/2} - \frac{\pi^2\,(1+bf_1/2f_2)}{2}\left[\frac{1}{4\pi^2(1+bf_1/2f_2)} + 1\right] r\,. \quad (59)$$

Here we note that working to leading order the consistency relation $n_s = n_s(r)$, becomes independent of the integration constants M_1 and χ_2 (11).

In Fig. 3 we show the evolution of the tensor-to-scalar ratio r w.r.t. the scalar spectral index n_s for three different values of the parameter α. Here we show the two-dimensional marginalized constraints, at 68 and 95 % levels of confidence, for the tensor-to-scalar ratio r and the spectral index n_s from BICEP2 experiment in connection with Planck + WP + highL [59]. In order to write down values that relate the ratio r and the spectral index n_s we considered the consistency relation $n_s = n_s(r)$ given by Eq. (59). Also, we have used the values $f_1 = 2 \times 10^{-8}$, $f_2 = 10^{-8}$, $\epsilon = 1$, $b = -0.52$ and $M_p = \sqrt{2}$.

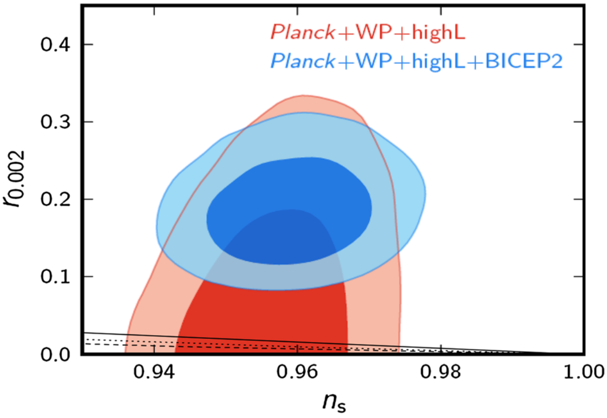

Fig. 3 Evolution of the tensor–scalar ratio r versus the scalar spectrum index n_s, for three different value of the parameter α. The *dashed*, *dotted*, and *solid lines* are for the values of $\alpha = 0.2$, $\alpha = 10^{-2}$ and $\alpha = 10^{-20}$, respectively. Also, in this plot we have taken the values $f_1 = 2 \times 10^{-8}$, $f_2 = 10^{-8}$, $\epsilon = 1$, $b = -0.52$ and $M_p = \sqrt{2}$

From the plot in Fig. 3 we note that the tensor-to-scalar ratio $r \sim 0$, and our model is disproved from BICEP2, since according to the latter the ratio $r = 0.2^{+0.07}_{-0.05}$ with the ratio $r = 0$ disproved at 7.0σ. Nevertheless, the result for tensor-to-scalar ratio has become less clear when serious criticisms of BICEP2 appeared in the literature. In particular, the Planck Collaboration has issued the data about the polarized dust emission through an analysis of the polarized thermal emissions from diffuse Galactic dust, which suggest that BICEP2 data of the gravitational wave result could be due to the dust contamination [60]. Thereby, a detailed analysis of Planck and BICEP2 data would be required for a definitive answer. In this form, previous CMB observations from the Planck satellite and other CMB experiments obtained only an upper limit for the tensor-to-scalar ratio, in which $r < 0.11$ (at 95 % confidence level) [61]. Therefore, we find that the value $\alpha \sim 0$ is well supported by the confidence levels from Planck data. In particular, the value $\alpha = 10^{-20}$ corresponds to $r\mid_{n_s=0.96} \simeq 0.017$. Also, we note that when we increase the value of the parameter $\alpha > 0.2$, the value of the tensor-to-scalar ratio $r \to 0$.

Besides, in particular for the values $\mathcal{P}_S \simeq 2.4 \times 10^{-9}$ and $N_* = 60$ (recall the subscript $*$ indicating the epoch where the cosmological scale exits the horizon) we obtained for the parameter χ_2 from Eq. (58) that $\chi_2 \simeq 74 \times 10^{-3}$, which corresponds to the value of $\alpha = 0.2$, and $\chi_2 \simeq 58 \times 10^{-6}$, which corresponds to the parameter $\alpha = 10^{-20}$. In this form the constraint for χ_2 is given by $58 \times 10^{-6} \lesssim \chi_2 \lesssim 74 \times 10^{-3}$. Here, we have used the same values of b, f_1, f_2 and ϵ from Fig. 3.

Numerically, from Eq. (55) we find a constraint for the parameter f_1 given by $f_1 \simeq 7.74 \times 10^{-8}$ for the values $n_s = 0.96$ and the number $N_* = 60$, which corresponds to the value of $\alpha = 0.2$, and $f_1 \simeq 3.58 \times 10^{-8}$, which corresponds to $\alpha = 10^{-20}$. In this way, the range of the parameter f_1 is $3.58 \times 10^{-8} \lesssim f_1 \lesssim 7.74 \times 10^{-8}$. As before, we have considered the same values of b, f_2 and ϵ from Fig. 3.

5 Non-singular emergent universe solution

We will now show that under appropriate restrictions on the parameters there exist an epoch preceding the inflationary phase. Namely, we derive an explicit cosmological solution of the Einstein-frame system with effective scalar field Lagrangian (24)–(27) describing a non-singular "emergent universe" [30–36] when the scalar field evolves on the first flat region for large negative φ (28). For previous studies of "emergent universe" scenarios within the context of the less general modified-measure gravity–matter theories with one non-Riemannian and one standard Riemannian integration measures, see Refs. [46–48].

Emergent universe is defined through the standard Friedman–Lemaitre–Robertson–Walker space–time metric (36) as a solution of (37) subject to the condition on the Hubble parameter H:

$$H = 0 \quad \rightarrow \quad a(t) = a_0 = \text{const}\,, \quad \rho + 3p = 0, \quad \frac{K}{a_0^2} = \frac{1}{6}\rho \;(= \text{const}), \tag{60}$$

with ρ and p as in (38), (39):

The emergent universe condition (60) implies that the φ-velocity $\dot{\varphi} \equiv \dot{\varphi}_0$ is time-independent and satisfies the bi-quadratic algebraic equation:

$$\frac{3}{2}B_{(-)}\dot{\varphi}_0^4 + 2A_{(-)}\dot{\varphi}_0^2 - 2U_{(-)} = 0 \tag{61}$$

(with notations as in (28), (29)), whose solution read:

$$\dot{\varphi}_0^2 = -\frac{2}{3B_{(-)}}\Big[A_{(-)} \mp \sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\Big]. \tag{62}$$

Let us note that according to (29) $B_{(-)} < 0$ for a wide range of the parameters, in particular, within the allowed interval of stability (see (69) below). We also observe that under the emergent universe condition (60), and since now $\dot{\varphi}$ is time-independent, the φ-equations of motion (41) are identically satisfied.

To analyze stability of the present emergent universe solution:

$$a_0^2 = \frac{6K}{\rho_0}, \quad \rho_0 = \frac{1}{2}A_{(-)}\dot{\varphi}_0^2 + \frac{3}{4}B_{(-)}\dot{\varphi}_0^4 + U_{(-)}, \tag{63}$$

with $\dot{\varphi}_0^2$ as in (62), we perturb Friedman Eqs. (37) and the expressions for $\rho,\ p$ (38), (39) w.r.t. $a(t) = a_0 + \delta a(t)$ and $\dot{\varphi}(t) = \dot{\varphi}_0 + \delta\dot{\varphi}(t)$, but keep the effective potential on the flat region $U_{\text{eff}} = U_{(-)}$:

$$\frac{\delta\ddot{a}}{a_0} + \frac{1}{12}(\delta\rho + 3\delta p), \quad \delta\rho = -\frac{2\rho_0}{a_0}\delta a, \tag{64}$$

$$\delta\rho = \left(A_{(-)}\dot{\varphi}_0 + 3B_{(-)}\dot{\varphi}_0^3\right)\delta\dot{\varphi} = -\frac{2\rho_0}{a_0}\delta a, \quad \delta p = \left(A_{(-)}\dot{\varphi}_0 + B_{(-)}\dot{\varphi}_0^3\right)\delta\dot{\varphi}. \tag{65}$$

From the first Eq. (65) expressing $\delta\dot{\varphi}$ as function of δa and substituting into the first Eq. (64) we get a harmonic oscillator type equation for δa:

$$\delta\ddot{a} + \omega^2 \delta a = 0, \quad \omega^2 \equiv \frac{2}{3}\rho_0 \frac{\pm\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}}{A \mp 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}}, \tag{66}$$

where:

$$\rho_0 \equiv \frac{1}{2}\dot{\varphi}_0^2\big[A_{(-)} + 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\big], \tag{67}$$

with $\dot{\varphi}_0^2$ from (62). Thus, for existence and stability of the emergent universe solution we have to choose the upper signs in (62), (66) and we need the conditions:

$$A_{(-)}^2 + 3B_{(-)}U_{(-)} > 0, \quad A_{(-)} - 2\sqrt{A_{(-)}^2 + 3B_{(-)}U_{(-)}}\big] > 0. \tag{68}$$

The latter yield the following constraint on the coupling parameters:

$$\max\Big\{-2\,,\; -8\big(1+3\epsilon f_1^2/f_2\big)\Big[1-\sqrt{1-\frac{1}{4\big(1+3\epsilon f_1^2/f_2\big)}}\Big]\Big\} < b\frac{f_1}{f_2} < -1, \tag{69}$$

in particular, implying that $b < 0$. The latter means that both terms in the original matter Lagrangian $L^{(2)}$ (4) appearing multiplied by the second non-Riemannian integration measure density Φ_2 (2) must be taken with "wrong" signs in order to have a consistent physical Einstein-frame theory (24)–(26) possessing a non-singular emergent universe solution.

For $\epsilon > 0$, since the ratio $\frac{f_1^2}{f_2}$ proportional to the height of the first flat region of the effective scalar potential, i.e., the vacuum energy density in the early universe, must be large (cf. (32)), we find that the lower end of the interval in (69) is very close to the upper end, i.e., $b\frac{f_1}{f_2} \simeq -1$.

From Eqs. (62), (63) we obtain an inequality satisfied by the initial energy density ρ_0 in the emergent universe: $U_{(-)} < \rho_0 < 2U_{(-)}$, which together with the estimate of the order of magnitude for $U_{(-)}$ (35) implies order of magnitude for $a_0^2 \sim 10^{-8} K M_{Pl}^{-2}$, where K is the Gaussian curvature of the spacial section.

For a recent semiclassical analysis of quantum (in)stability of oscillating emergent universes we refer to [65–67].

6 Evolution of the universe to its present slowly accelerating state

As a first approach to an unified analysis of all stages of the cosmological scenario developed here (emergent universe, transition from emergent universe to inflation, slow-roll regime, etc.) we write the set of dynamical equations (37) and (41) as an autonomous system of three dimensions by following the scheme developed in Ref. [46] (for a recent systematic exposition of the methods of dynamical systems' evolution in the context of cosmology, see [68]). We obtain:

$$\dot{H} = -H^2 + \frac{1}{12}\left(2A(\varphi)\,x^2 + \frac{3}{2}B(\varphi)\,x^4 - 2U_{eff}\right), \tag{70}$$

$$\dot{x} = -\frac{3H\,x\,(A(\varphi) + B(\varphi)\,x^2) + \frac{1}{2}A'\,x^2 + \frac{3}{4}B'\,x^4 + U'_{eff}}{A(\varphi) + 3B(\varphi)\,x^2}, \tag{71}$$

$$\dot{\varphi} = x, \tag{72}$$

where we have defined $x = \dot{\varphi}$. We are considering $\dot{\varphi} > 0$ because we are interested in the cases where the field φ moves from $-\infty$ to positives values, following the emergent universe scheme.

During the emergent universe regime the scalar field evolves on the first flat region (28) of the effective potential corresponding to large negative φ. In this case the set of equations (70)–(72) could be written as an autonomous system of two dimensions with respect to H and x as follows:

$$\dot{H} = -H^2 + \frac{1}{12}\left(2A_{(-)}x^2 + \frac{3}{2}B_{(-)}\,x^4 - 2U_{(-)}\right), \tag{73}$$

$$\dot{x} = -\frac{3H\,x\,(A_{(-)} + B_{(-)}x^2)}{A_{(-)} + 3B_{(-)}x^2}. \tag{74}$$

For $x > 0$ the above system has six critical points. In order to study the nature of these critical points we linearize the set of equations (73), (74) near the critical points. From the study of the eigenvalues of the system and by taking into account the constraints on the pertinent parameters (68), (69) discussed in the previous Sect. 5 we find that two of these critical points correspond to the emergent universe solution, where $H_0 = 0$:

$$x_0^2 = \frac{8b\,f_1\chi_2 + 16f_2\chi_2 \mp \sqrt{(-8bf_1\chi_2 - 16f_2\chi_2)^2 - 16f_1^2\left(3b^2\chi_2^2 - 12bf_1\epsilon\chi_2^2 - 12f_2\epsilon\chi_2^2\right)}}{2\left(3b^2\chi_2^2 - 12bf_1\epsilon\chi_2^2 - 12f_2\epsilon\chi_2^2\right)},$$
$$H_0 = 0. \tag{75}$$

In this case, the upper sign in Eq. (75) corresponds to a center critical point and the lower sign is an unstable saddle point. The stable emergent universe solution obtained in Sect. 5 is the stable center critical point.

Also, we have the following critical points which are (upper sign) attractor and a (lower sign) focus:

$$x_0^2 = \frac{2(bf_1 + 2f_2)}{\left(b^2 - 4bf_1\epsilon - 4f_2\epsilon\right)\chi_2}, \tag{76}$$

$$H_0 = \pm\frac{\sqrt{\frac{2f_1^2}{(f_2+f_1^2\epsilon)\chi_2} - \frac{4(bf_1+2f_2)}{(b^2-4bf_1\epsilon-4f_2\epsilon)\chi_2} - \frac{2bf_1(bf_1+2f_2)}{(f_2+f_1^2\epsilon)(b^2-4bf_1\epsilon-4f_2\epsilon)\chi_2} + \frac{4f_1^2(bf_1+2f_2)\epsilon}{(f_2+f_1^2\epsilon)(b^2-4bf_1\epsilon-4f_2\epsilon)\chi_2}}}{4\sqrt{3}}. \tag{77}$$

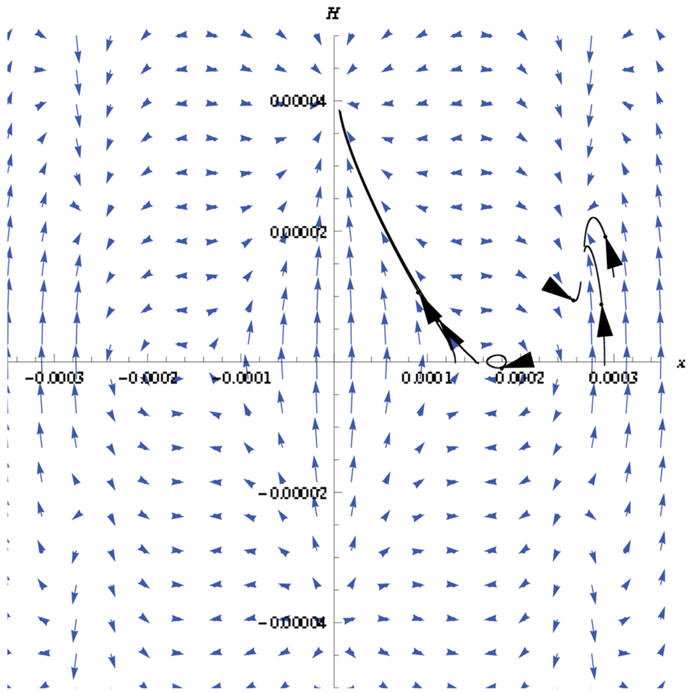

Fig. 4 Plot showing part of the direction field of the system Eqs. (73), (74) and six numerical solutions

These critical points are similar to the kinetic vacuum state discussed in Ref. [46], but in the present case these critical points only exists in the limit $\varphi \to -\infty$.

On the other hand, we have the standard slow-roll de Sitter critical point:

$$x_0^2 = 0, \qquad H_0 = \pm \frac{f_1}{2\sqrt{6}\sqrt{f_2 + f_1^2 \epsilon \sqrt{\chi_2}}}, \tag{78}$$

where the upper sign is an attractor and the lower sign is a focus in Eq. (78). This is the standard slow-roll de Sitter attractor.

In Fig. 4 it is shown a phase portrait for six numerical solution to Eqs. (73), (74), where we have taken $f_1 = 2 \times 10^{-8}$, $f_2 = 1 \times 10^{-8}$, $\epsilon = 1, \alpha = 1, \chi_2 = 1, M_1 = 4 \times 10^{-60}$, $M_2 = 4$, and $b = -0.52$. Also, in this figure we have included the direction field of the system in order to have a visual picture of what a general solution look like. In Fig. 4 the six critical points described above are depicted. One of this points is the center equilibrium point $(H = 0, x = 0.00017)$, the saddle point $(H = 0, x = 0.00025)$, the point $(H = 0.000015, x = 0.00026)$ is a future attractor and $(H = -0.000015, x = 0.00026)$ is a past attractor. The other equilibrium points are $(H = 0.000040, x = 0)$ and $(H = -0.000040, x = 0)$ which are a future attractor point and a past attractor point, respectively.

As we have mentioned above, the slow-roll de Sitter critical point is an attractor, then, it is plausible that some of the solutions near the center critical point,

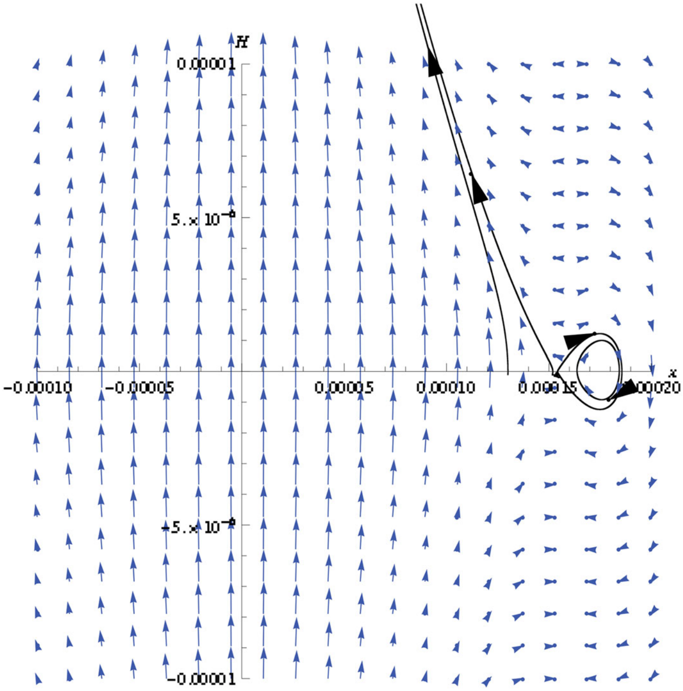

Fig. 5 Plot showing part of the direction field of the system and four numerical solutions, near the center critical point

when the effective potential begins to be nonconstant, start to move away from the center critical point and finish at the slow-roll de Sitter critical point; see Fig. 5. During this short period, which occurs before the slow-roll period, the Hubble parameter satisfies $\dot{H} > 0$. This period is called "super-inflation" and has been studied in the context of emergent universe scenario in Ref. [69]. After this short period the system arrives at the slow-roll regime discussed in Sect. 3 above.

On the other hand, during the present slowly accelerating phase of the universe, the scalar field evolves on the second flat region of the effective potential (30) corresponding to large positive φ.

In this case the set of Eqs. (70)–(72) can be written as an autonomous system of two dimensions as follows:

$$\dot{H} = \frac{M_1^2\left(1 - 24H^2\epsilon\,\chi_2\right) - M_2\chi_2\left(24H^2 + 4x^2 + 3x^4\epsilon\,\chi_2\right)}{24\left(M_2 + M_1^2\,\epsilon\right)\chi_2}\,, \tag{79}$$

$$\dot{x} = -\frac{3Hx\left(1 + x^2\epsilon\,\chi_2\right)}{1 + 3x^2\epsilon\chi_2}\,. \tag{80}$$

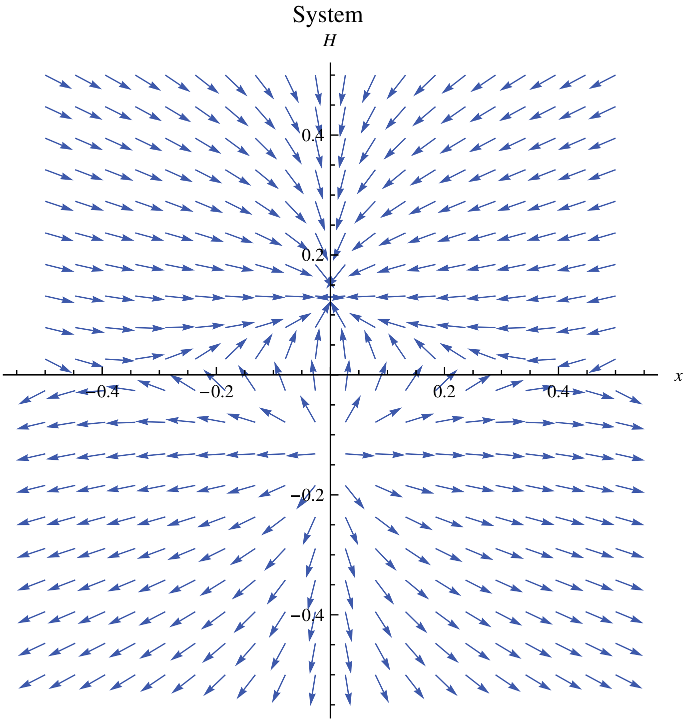

Fig. 6 Plot showing part of the direction field of the system Eqs. (79), (80)

The system has two critical points:

$$H_0 = \pm \frac{M_1}{2\sqrt{6}\sqrt{M_2 + M_1^2 \epsilon \sqrt{\chi_2}}}\,, \tag{81}$$

$$x_0 = 0\,. \tag{82}$$

These critical points are an attractor and a focus respectively. In Fig. 6 it is shown the qualitative shape of the direction field of the system Eqs. (79), (80) near the critical points. In particular, we have considered the values $M_1 = 2$ and $M_2 = 6$ instead of the values considered previously, in order to have a more clear view of the critical points in the direction field. In this figure there are the two critical points described above. One of this point is the future attractor ($H = 0.129$, $x = 0$), and the other is the past attractor point ($H = -0.129$, $x = 0$).

7 Discussion

In the present paper we have constructed a new kind of gravity–matter theory defined in terms of two different non-Riemannian volume-forms (generally covariant integration measure densities) on the space–time manifold, where the Einstein–Hilbert term R,

Table 1 Results for the constraints on the parameters in our model

Phase	Constraint from	Constraint on
Dark energy dominated	Vacuum energy density Eq. (34)	$\frac{M_1^2}{M_2} \simeq 10^{-120} M_{Pl}^4$
Inflation (using also emergent)	Eq.(35) $\mathcal{P}_S \simeq 2.4 \times 10^{-9}$and $N_* = 60$ $n_s = 0.96$ and $N_* = 60$ Consistency relation $n_s = n_s(r)$	$\frac{f_1^2}{f_2} \sim 10^{-8} M_{Pl}^4$ $58 \times 10^{-6} \lesssim \chi_2 \lesssim 74 \times 10^{-3}$ $3.6 \times 10^{-8} \lesssim f_1 \lesssim 7.7 \times 10^{-8}$ $0 \lesssim \alpha \lesssim 0.2$
Non-singular emergent	Upper end of the interval in Eq. (69)	$b \frac{f_1}{f_2} \simeq -1$

its square R^2, the kinetic and the potential terms in the pertinent cosmological scalar field (a "dilaton") couple to each of the non-Riemannian integration measures in a manifestly globally Weyl-scale invariant form. The principal results are as follows:

- Dynamical spontaneous symmetry breaking of the global Weyl-scale invariance.
- In the physical Einstein frame we obtain an effective scalar field potential with *two flat regions*—one corresponding to the early universe evolution and a second one for the present slowly accelerating phase of the universe.
- The flat region of the effective scalar potential appropriate for describing the early universe allows for the existence of a *non-singular "emergent"* type beginning of the universe' evolution. This "emergent" phase is followed, via a short period of "super-inflation", by the inflationary phase, which in turn is followed by a period, where the scalar field drops from its high energy density state to the present slowly accelerating phase of the universe.
- For a reasonable choice of the parameters the resulting ratio of tensor-to-scalar perturbations conforms to the data of Planck Collaboration.

The flatness of the effective scalar potential in the high energy density region makes the slow rolling inflation regime possible.

The presence of the emergent universe' phase preceding the inflationary phase has observable consequences for the low CMB multipoles as has been recently shown in Ref. [69].

Table 1 summarizes the constraints on the parameters in the different phases in the context of the present model. Let us note that although we don't have separate constraints on M_1 and M_2, nevertheless on Sect. 3 we made the natural choice to identify them with the two fundamental scales M_{EW} and M_{Pl}, which then yielded the correct order of magnitude (34) of the present epoche's dark energy dominated vacuum energy density. Similarly, although the inflationary phase only determines the scale of the ratio f_1^2/f_2 (35) we made the natural choice for these parameters setting $f_1 \sim f_2$ to be of the same order of magnitude since both originally appear as coupling constants in front of two scalar field potential terms of the same type. For the last parameter ϵ we have found the restriction $|\epsilon|\frac{M_1^2}{M_2} \ll 1$ (second inequality in (33)).

We conclude with some comments of qualitative nature. The oscillations of the scalar field φ are an important part for the standard mechanism of reheating of the

Universe [70]. However, when the integration constant $M_1 < 0$ our effective scalar potential (27) does not have a minimum (cf. Fig. 1) so that the scalar field φ does not oscillate and, therefore, the standard reheating does work. In the literature these models are known as non-oscillating (NO) models [71]. An option for the mechanism of reheating in these NO models is the instant preheating which inserts an interaction between the scalar field driving the inflationary scenario and another scalar field [72]. Other mechanism of reheating for the NO models is the insertion of the curvaton field [73]. Here the decay of the curvaton into conventional matter offers an effective reheating scenario, and does not introduce an interaction between the inflaton field and another scalar field [74,75]. In a future work we will study the extension of the present model to include a curvaton field according to the basic principles of two non-Riemannian volume forms on the underlying spacetime and of spontaneous breakdown of global Weyl-scale invariance.

When the integration constant $M_1 > 0$ the effective scalar potential (27) possesses an absolute minimum $U_{\rm eff} = 0$ at $\varphi = \varphi_{\rm min}$, where $\exp\{-\alpha\varphi_{\rm min}\} = M_1/f_1$ (cf. Fig. 2). As it is evident from Fig. 2, there is an abrupt fall to $U_{\rm eff} = 0$ where particle creation will take place when we consider the extended theory enlarged with a curvaton as mentioned above. The scalar field falls down with very high kinetic energy into the region of $U_{\rm eff} \simeq 0$, the kinetic energy before the fall-down being certainly vastly higher than the value of $U_{\rm eff} \simeq U_{(+)}$ (34) in the second flat region to the right. So $\varphi(t)$ "climbs" the latter very low barrier and continues to evolve in the $\varphi \to +\infty$ direction. Thus, on the second flat region we have a slow rolling scalar field which produces approximately the dark energy equation of state $\rho \simeq -p$, with very small $\rho = U_{(+)}$ (34) explaining the present day dark energy phase. In a future work we plan to study in more details the evolution of the scalar field in the vicinity of, and its escape out of, the global minimum $U_{\rm eff} = 0$.

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